Effect of Implied Volatility on Option Prices using two Option Pricing Models

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Abstract

This paper analyses the Black-Scholes and Heston Option Pricing Model. We discuss the concept of implied volatility in the two models. We compare the two models for the parameter 'Volatility'. A mathematical tool, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is used to compare the two models for live implied volatility while pricing European call options. Real data from NSE (National Stock Exchange) is considered for three different sectors - Banking, Automobiles and Pharmaceuticals - for comparison through 'Moneyness' (which is defined as the percentage difference of stock price and strike price) and Time-To-Maturity. Mathematical software – Matlab is used for all mathematical work.

Keywords: *European call option, Black-Scholes model, Heston Model, Moneyness, Time-to-maturity, Implied Volatility*

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Introduction

Financial derivatives have been drawing increasing interest in recent days. Among these, Options are the most basic and fundamental derivates. European options are most widely used in Indian stock exchanges. There are different types of options available for pricing; Black-Scholes is one of such type. Black-Scholes model is used for pricing options to calculate the premium value.

In the early 1960's, many mathematicians such as Sprekle, Ayes, A. James Boness, Chen etc. worked on the valuation of options. In 1973, Fischer Black and Myron Scholes developed the options pricing formula, which later made use of partial differential equation with coefficient variables. It uses historical volatility as a measure of calculation with various assumptions. Yuang (2006) used the concept of implied volatility. In 1993, Heston proposed a stochastic volatility model. It used the assumption that the asset variance v_t follows a mean reverting Cox-Ingersoll-Ross process.

The Black-Scholes model assumes that volatility remains constant through the option's life which is not practical with the real fluctuating market. The Heston model removed such assumptions and is more favourable to the financial market. Stochastic Volatility removes excess kurtosis and asymmetry.

UMBRAE (Unscaled Mean Bounded Relative Absolute Error) (Chen et al., 2017) is a measure of error calculation in the model. It is helpful in removing symmetric and bounded error during forecasting. Here, we use Naive method as the benchmark for forecasting UMBRAE. We observe the performance of Heston Model and Black-Scholes Model in three different sectors - Banking, Pharmaceuticals and Automobiles - under parameters like time-to-maturity and moneyness (which is defined as the percentage difference of stock price and strike price) while pricing European call options.

Volatility is the most important factor of options trading. There are many types of volatility in the market. Historical and Implied Volatility are primarily used for options pricing.

Historical volatility is the annualized standard deviation of the past stock data. It measures the price change in the stock over the year.

Implied volatility is derived from Options model formula. It shows the future probability of a volatile market.

This paper is organised as follows; starting with the basic concepts of Black-Scholes model, we discuss all the parameters of the model followed by sensitivity analysis of the model. Second, we discuss the Heston Model, Implied Volatility, an accuracy measure UMBRAE, and Methodology of the models. Finally, the two models are compared for different Indian stock data.

In this paper, we will answer the following questions:

i. How does Implied volatility work in two different option pricing models?

ii. Is Black-Scholes model different from Heston model in different sectors of stocks?

iii. How does moneyness play a vital role in option pricing?

Literature Review

In 1960's, work on financial analysis was started by Sprenkle (1961), Ayes (1963), A.James Boness (1964), Samuelson (1965), Baumol, Malkiel and Quandt (1966), Chen (1970) etc. The options pricing formula for European Options was provided by Black F. and Scholes M. (1973). This gives the theoretical value of options pricing, which is also helpful in corporate bonds and warrants. However, Black-Scholes model uses various assumptions for calculations, which are not accurate in the practical world.

Shinde A. (2012) explains the basic terminologies of the Black-Scholes (B-S) options pricing model in a familiar and easy way. Kalra (2015) studied the effect of volatility in different economic situations of the Indian stock market. Singh Gurmeet (2015) attempted to model the volatility of Nifty index and showed that Arch models outperform OLS models. He analysed that ARIM(1,0,1) model was the best fit in the short run Nifty stock returns.

Further, different mathematicians worked on modified models for improvement on the initial model. Singh and Gor (2020a) studied the B-S options pricing model and the model where underlying stock returns follow the Gumbel distribution at maturity, and compared the result for actual market data. Singh and Gor (2020b) also compared the B-S model to a different model where stock returns follow truncated Gumbel distribution. Chauhan and Gor (2020b) studied the modified truncated Black-Scholes model and compared the result with the original B-S model.

Modified B-S model works better than the original B-S model. However, another model which uses volatility as stochastic quantity was introduced in 1993. Heston (1993) proposed a stochastic volatility model for European Call options. It considers volatility as the stochastic quantity, while Black-Scholes considered volatility as constant. The stochastic volatility model works better in the real-world market while removing excess of skewness and kurotsis in the model. It is also much more accurate in theoretical premium values.

Crisostomo R. (2014) and Yuan Yang (2013) derived the Heston characteristic function and formula for call value and compared the Black-Scholes and Heston option pricing formula for different parameters. They presented a graphical comparison among different parameters of option pricing model. Ziqun Ye (2013) introduced the concept of moneyness and compared the two models for different parameters of moneyness and Time-To-Maturity.

Santra A. (2017) used Matlab software for calculating theoretical call value of Black-Scholes and Heston models. He also provided a detailed explanation and compared for different options of moneyness. Chen et al. (2017) provided a new accuracy measure for calculating error for forecasting methods. We have used this accuracy measure to compare the two options pricing models in real market data for different options of moneyness and time-to-maturity. Sisodia and Gor (2020) also worked on estimating the relevance of option pricing models for European call-put options. They have compared the B-S model and Heston model for selective stocks and analysed the result for different options of moneyness and time-to-maturity.

Basic Concepts [7]

a. **Option:** An option is defined as the right, but not the obligation, to buy (call option) or sell (put option) a specific asset by paying a strike price on or before a specific date.
    (I) *Call option:* An option which grants its holder the right to buy the underlying asset at a strike price at some moment in the future.
    (ii) *Put option:* An option which grants its holder the right to sell the underlying asset at a strike price at some moment in the future.
b. **Expiration Date/ Time-to-maturity:** The date on which an option right expires and becomes worthless if not exercised. In European options, an option cannot be exercised until the expiration date.
c. **Strike Price:** The predetermined price of an underlying asset is called strike price.
d. **Stochastic Process**: Any variable whose value changes over time in an uncertain way is said to follow a stochastic process.
e. **Stochastic Volatility:** Volatility is a measure for variation of price of a stock over time. Stochastic in this sense refers to successive values of a random variable that are not independent.
f. **Geometric Brownian Motion:** A continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion.
g. **Moneyness:** It is the relative position of the current price of an underlying asset with respect to the strike price of a derivative, most commonly a call/put option.

h. **Black-Scholes Inputs / Parameters:**

There are six basic parameters used in pricing an option in Black-Scholes model.

They are as follows:

- Underlying stock price
- Strike price
- Time to expiration
- Interest rate
- Volatility

Volatility - It is the standard deviation of the continuously compounded return of the stock. In other words, we can say that volatility reflects fluctuations in the market. It is one of the important variables of options pricing. For both Call and Put options, options' price increase as volatility increases. There are mainly two types of volatility in the market – Historical and Implied.

Historical volatility is calculated from past data in the market. Implied volatility is derived from options' prices or options' pricing model. It is also available on a daily basis on the website of stock exchanges. It is denoted by the symbol σ (sigma), in the model formula.

Implied Volatility: Implied Volatility is a metric that captures the market's view of the likelihood of changes in a given security's price. The option's premium price component changes as the expectation of volatility changes over time. It helps to predict future market fluctuations. It shows the move of the market, but not the direction. It is denoted by the symbol σ (sigma), commonly expressed as standard deviation over time. High implied volatility results in options with higher value, and vice-versa.

The Black-Scholes Model [16]

This model is based on the following assumptions:

- Stock pays no dividends.
- Option can only be exercised upon expiration.
- Random walk.
- No transaction cost.
- Interest rate remains constant.
- Stock returns are normally distributed; thus, volatility is constant over time.

In 1973, Fischer Black and Myron Scholes proposed a model for European Call options based on Geometric Brownian motion.

$$\mathrm{dS_t = \mu S_t dt + \sigma_t S_t dW_t}$$

Where, S_t is the price of the asset, μ is the drift (constant), σ_t is the return volatility(constant) and W_t is the Brownian motion. Black[1] showed that we can use the risk neutral probability rather than the true probability to evaluate the price of an option.

The risk neutral dynamics on an asset is given by:

$$dS_t = rS_t dt + \sigma_t S_t dW_t$$

Where, r is the risk-free rate.

The solution to the above stochastic differential equation is a Geometric Brownian Motion:

$$S_t = S_0\, exp\left[\sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right)t\right]$$

The log of which is a Geometric Brownian Motion (GBM) model for stock prices.

$$ln\left(\frac{S_t}{S_0}\right) = \sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right)t$$

Where, R.H.S. equation is a normal random variable whose mean is $\left(\mu - \frac{\sigma^2}{2}\right)t$ And variance is $\sigma^2 t$.

The Black-Scholes formula for European call price is,

$$C = S_0\, N(d_1) - Ke^{-rt}N(d_2)$$

Where $d_1 = \frac{ln(S_0/K) + (r+\sigma^2/2)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$

K is the strike price, S_0 is today's stock price, *t* is time to expiration, *r* is riskless interest rate (constant), σ is volatility of stock (constant).

The Heston Model [9]

In 1993, Heston proposed a Stochastic Volatility Model. Consider at time t the underlying asset S_t which obeys a diffusion process with volatility being treated as a latent stochastic process of Feller as proposed by Cox, Ingersoll and Ross:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1$$

$$dV_t = k[\theta - V_t]dt + \sigma\sqrt{V_t} dW_t^2$$

Where, W_t^1 and W_t^2 are two correlated Brownian motion with a correlation coefficient given by $\rho > 0$:

$$dW_t^1 dW_t^2 = \rho dt$$

Where, S_t is the price of the asset, *r* is the risk-free rate, V_t is the variance at time *t*, $\theta > 0$ is the long term mean variance, $k > 0$ is variance mean-reversion speed, $\sigma \geq 0$ is the volatility of the variance.

The price of a European call option can be obtained by using the following equation:

$$C = S_0 \Pi_1 - e^{-rt} K \Pi_2$$

Where, Π_1 is the delta of the option and Π_2 is the risk-neutral probability of exercise (i.e. when $S_t > K$)

For j=1, 2 the Heston characteristic function is given as:

$$f_j(x, v, \tau; \emptyset) = e^{C(\tau;\emptyset) + D(\tau;\emptyset)v + i\emptyset x}$$

Where,

$$C(\tau;\emptyset) = r\emptyset i\tau + \frac{a}{\sigma^2}\left\{(b_j - \rho\sigma\emptyset i + d)\tau - 2ln\left[\frac{1 - ge^{d\tau}}{1 - g}\right]\right\}$$

$$D(\tau;\emptyset) = \frac{b_j - \rho\sigma\emptyset i + d}{\sigma^2}\left[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}\right]$$

$$g = \frac{b_j - \rho\sigma\emptyset i + d}{b_j - \rho\sigma\emptyset i - d}$$

$$d = \sqrt{(\rho\sigma\emptyset i - b_j)^2 - \sigma^2(2u_j\emptyset i - \emptyset^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = k\theta, b_1 = k - \rho\sigma, b_2 = k$$

The characteristic functions can be inverted to get the required probabilities.

$$\Pi_j(x, v, T; \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\emptyset \ln[K]} f_j(x, v, T; \emptyset)}{i\emptyset}\right] d\emptyset$$

Methodology

DATA:

The data has been collected for call options from Banking, Pharmaceuticals and Automobile sectors from the website of National stock Exchange of India. Call options data for the following stocks were considered:

For <u>Banking</u> – Axis Bank, Federal Bank, HDFC Bank and Kotak Mahindra Bank

For <u>Automobile companies</u> – Tata Motors, TVS Motors, Maruti Udyog, Hero Honda Motors and Mahindra & Mahindra

For <u>Pharmaceutical companies</u> - Sun Pharmaceuticals, Lupin limited, Dr. Reddy's Laboratories, Cipla Limited and Zydus Cadila Healthcare Limited.

The period from November 22 to November 30, 2018 has been considered.

PARAMETERS:

The option moneyness is defined as the percentage difference between the current underlying price and the strike price:

- Moneyness(%) = S / K-1

  The result has been divided in terms of moneyness and time-to-maturity.

- ATM (At the money) – A call option is at the money if the strike price is the same as the current underlying stock price.
- ITM (In the money) – A call option is in the money when the strike price is below the underlying stock price.
- OTM (Out of the money) – A call option is out of the money when the strike price is above the underlying stock price.
- UMBRAE (Unscaled Mean Bounded Relative Absolute Error) = $\frac{\text{MBRAE}}{1-\text{MBRAE}}$

$$MBRAE = \frac{1}{n}\sum_{t=1}^{n}(BRAE)$$

$$BRAE = \frac{|e_t|}{|e_t| + |e_t^*|}$$

$$e_t = y_t - f_t$$

$$e_t^* = y_t - f_t^*$$

Where, y_t is the observed model price, f_t is the actual market forecasted value and, f_t^* is the market forecasted value using the Naive Method.

UMBRAE has a lower bound of 0 and an upper bound of 1.

We have used Matlab function *bsm_price* and run the model to calculate the European call option value using the following parameters (Binay and Santra, 2017):

- Risk-free interest rate: It is the rate at which we deposit or borrow cash over the life of the option. Call option value increases as the risk-free rate increases. It takes value 0.05 throughout the function.
- Volatility: It is the standard deviation of the continuously compounded return of the stock. Call option value is directly correlated to volatility, i.e. higher the volatility, higher the call option value.

We have used Matlab function *heston_chfun* for the Heston characteristic function and *heston_price* for the calculation of European call option value using the following parameters (Binay and Santra, 2017):

- Initial Variance: Bounds of 0 and 1 have been used.
- Long-term Variance: Bounds of 0 and 1 have been used.
- Correlation: Correlation between the stochastic processes takes values from -1 to 1.
- Volatility of Variance: It exhibits positive values. Since the volatility of assets may increase in the short term, a broad range of 0 to 5 will be used.
- Mean-Reversion Speed: This will be dynamically set using a non-negative constraint (Feller, 1951). The constraint $2k\theta - \sigma^2 > 0$ guarantees that the variance in CIR process is always strictly positive.
- Initial Variance = 0.28087
- Long-term Variance = 0.001001
- Volatility of Variance = 0.1
- Correlation Coefficient = 0.5
- Mean Reversion Speed = 2.931465

Result and Analysis

UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is calculated for different options of moneyness and time-to-maturity using Implied volatility in each sector between Black-Scholes and Heston option pricing model.

Table 1 - Axis Bank

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=560	ATM, K=610	OTM, K=690
Black-Scholes	0	0.09	0.09
Heston	0	3.72	1.1

From Table 1, we observe that Black-Scholes model outperforms Heston Model for all ATM and OTM options of moneyness giving lesser error value. ITM option has the same impact on both the models as the error value is zero.

Table 2 - Federal Bank

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=72.50	ATM, K=80	OTM, K=105
Black-Scholes	0.11	0	0.06
Heston	0.89	0.36	0.25

From Table 2, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 3 - HDFC Bank

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=1660	ATM, K=1860	OTM, K=2200
Black-Scholes	0.01	0.01	1.19
Heston	0.83	1.54	8.55

From Table 3, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 4 - Kotak Mahindra Bank

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=1120	ATM, K=1160	OTM, K=1300
Black-Scholes	0	0	0.01
Heston	2.91	2.91	2.14

From Table 4, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 5 - Cadila Healthcare limited

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=350	ATM, K=360	OTM, K=440
Black-Scholes	0	0.78	0
Heston	0.78	0.01	0.03

From Table 5, we observe that Black-Scholes Model outperforms Heston Model for ITM and OTM options of moneyness giving lesser error value, while Heston Model outperforms better in ATM options.

Table 6 - Cipla Limited

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=500	ATM, K=520	OTM, K=650
Black-Scholes	16.2	0.01	0.09
Heston	2.09	3.29	0.57

From Table 6, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 7 - Lupin Pharmaceuticals limited

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=800	ATM, K=840	OTM, K=1040
Black-Scholes	0	0	0.25
Heston	0.22	2.25	2.11

From Table 7, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 8 - Dr. Reddy's laboratory

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=2450	ATM, K=2650	OTM, K=3000
Black-Scholes	0	0.01	0.01
Heston	0.76	2.29	2.21

From Table 8, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 9 - Sun Pharmaceuticals limited

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=480	ATM, K=530	OTM, K=700
Black-Scholes	0.05	0.06	0.01
Heston	0.05	2.12	0.07

From Table 9, we observe that Black-Scholes Model outperforms Heston Model for ATM and OTM options of moneyness giving lesser error value, while ITM option shows no difference in the two models.

Table 10 - TVS Motors

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=520	ATM, K=530	OTM, K=620
Black-Scholes	0	0.01	0.06
Heston	0.23	3.03	4

From Table 10, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 11 - TATA Motors

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=150	ATM, K=185	OTM, K=320
Black-Scholes	0.02	0.01	0.17
Heston	2.55	9	0.5

From Table 11, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 12 - Mahindra & Mahindra

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=730	ATM, K=750	OTM, K=900
Black-Scholes	0.01	0.01	0
Heston	2.47	1.77	0.03

From Table 12, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 13 - Maruti Udyog Limited

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=6500	ATM, K=7300	OTM, K=9900
Black-Scholes	0.01	0.05	0.06
Heston	0.37	2.25	2.95

From Table 13, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Table 14 - Hero Motor Corp

Models	Error Value at different strike prices (K) for option moneyness		
	ITM, K=2750	ATM, K=2900	OTM, K=3200
Black-Scholes	0	0.07	0.28
Heston	0.79	2.06	1.05

From Table 14, we observe that Black-Scholes Model outperforms Heston Model for all three options of moneyness giving lesser error value.

Applicability and Generalizability

The Black-Scholes model is widely used in the Indian market for prediction of theoretical premium value of stock options. The Heston model uses stochastic volatility which is more practical to market conditions and also free from excess skewness and kurtosis that appears in Black-Scholes model. Thus, comparing the two models, we have focussed on the performance of the models for different moneyness options. We have analysed it for three main sectors of the market, which will be helpful and generalized for further theoretical study of premium values of stock options.

Conclusion

We observe the following in case of the Banking sector:
In case of Out-of-the-money and At-the-money options, Black-Scholes Model outperforms Heston model for the chosen banks. For In-the-money option, only in case of Axis Bank we get the same result for both the models. In all other cases, Black-Scholes model gives better results.

We observe the following in case of the Automobile sector:
Black-Scholes outperforms the Heston model in all the companies for all the three cases - In-the-money, Out-of-the-money and At-the-money options.

We observe the following in case of the Pharmaceuticals sector:
For Out-of-the-money option, Black-Scholes gives better results than Heston Model for the chosen companies. For At-the-money option, Heston Model outperforms Black-Scholes' Model for Cadila Healthcare Limited. In the rest of the cases, Black-Scholes gives better results. For In-the-money option, Heston model outperforms Black-Scholes model for Cipla Pharmaceuticals. In case of Sun Pharma, both the models give the same result. For the rest of the companies, Black-Scholes gives better results.

As per the data chosen, we conclude that if we consider Implied volatility in the calculation of theoretical value of European Call Option, Black-Scholes Model outperforms Heston model in most cases in all the three options of moneyness for Banking, Automobile and Pharmaceutical sectors. This study is more helpful for derivative investors for short term and long term options. Mathematical models are always helpful in calculating theoretical premium values and this quantitative study of models could be extended in future for a large data set for more accurate results and suggestions.

References

- Black F. and Scholes M. (1973). “The Pricing of Options and Corporate Liabilities”, Journal of Political Economy, 81(3):637-644.
- Chauhan Arun, and Ravi Gor, (2020b). “A Comparative Study of Modified Black-Scholes option pricing formula for selected Indian call options”, IOSR Journal of Mathematics, 16(5),16-22.
- Chauhan Arun, and Ravi Gor, (2020). “Study of India VIX Options pricing using Black-Scholes model”, NMIMS Management Review, 38(4): 1-15.
- Chao Chen, Jamie T. and Jonathan M. (2017). “A new accuracy measure based on bounded relative error for time series forecasting”, PLoS ONE 12(3): e0174202, 6-7.
- Crisostomo R. (2014). “An Analysis of the Heston Stochastic Volatility Model: Implementation and Calibration using Matlab”, CNMV working paper No.58, Madrid,6-14.
- Heston S.L. (1993). “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options”, The Review of Financial Studies, 6(2):327-343.
- Hull J. (2009). “Options, Futures and other Derivatives”, Pearson Publication, Toronto.
- Karla, Rosy. (2015). “Volatility patterns of stock returns in India”, NMIMS Management Review, 27: 1-8.
- Santra A. and Binay B. (2017). “Comparison of Black-Scholes and Heston Models for Pricing Index Options”, Indian Institute of Management, Calcutta, W.P.S. No. 796,2-6.
- Shinde A.S. and Takale K.C. (2012). “Study of Black-Scholes Model and its Applications”, Procedia Engineering, (38):270-279.
- Singh, Akash, Gor Ravi. (2020a). “Relevancy of pricing European put option based on Gumbel distribution in actual market”, Alochana Chakra Journal, 9 (6): 4339-4342.
- Singh, Akash, Gor Ravi. (2020b). “Relevancy of pricing European put option based on truncated Gumbel distribution in actual market”, IOSR Journal of Mathematics, 16 (5): 12-15.
- Singh Gurmeet (2015). “Volatility Modelling and Forecasting for NIFTY stock returns”, NMIMS Management Review 27 (3): 89-108.
- Sisodia Neha and Gor Ravi (2020). “Estimating the relevancy of two option pricing models”, Alochana Chakra Journal, 9(6):5208-5211.
- Yuang Y. (2013). “Valuing a European Options with the Heston Model”, Thesis, Rochester Institute of Technology, 25-28. https://scholarworks.rit.edu/theses/4809
- Ziqun Ye. (2013). “The Black-Scholes and Heston Models for Option Pricing”, Waterloo University, Ontario, UW Space, 23-25. http://hdl.handle.net/10012/7541

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